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# Growth and the Enlargement of a Common Market

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*This paper explores the growth effects of the enlargement of a common market from two to three countries by making use of a three-country equilibrium growth model with heterogeneous labour. We prove that the enlargement will stimulate the backward countries' economic growth. In addition, we also demonstrate that the higher the new member country's average talent level is, the more likely it is that the enlargement can speed up the initial integrated-economy's economic growth.*

***Keywords:*** *diversity; common market; equilibrium growth; factor mobility*

***JEL classification****: F15; O41; R23*

## Introduction

Can diversity which refers to the dispersion of workers' talent or human capital, as discussed in Kremer (1993) and Grossman and Maggi (2000), stimulate economic growth? In the pioneering work, Das (2005) examines the growth effect of diversity in an equilibrium growth model where new product innovation, including blueprints or ideas, is driven by R&D sector, as derived by Romer (1990) and Jones (2005). Das shows that diversity could speed up economic growth for a closed economy.

In the type of deepening economic integration, Walz (1998) analyzes the growth effect of a enlargement of a common market referring to dismantling barriers to factor movements among the member countries and shows that relaxing barriers to migration for unskilled labor or emigration for skilled labor, from the initial integration bloc point of view, might lead to a reduction in growth. However, Walz doesn't embody the characteristics of heterogeneous or diverse workers. That is to say, by embracing with heterogeneous human capital assumption, the impact of the enlargement of a common market on growth is not discussed. Therefore, in this paper, we intend to fill this gap.

We construct a three-country, two-sector equilibrium growth model with heterogeneous labor, to analyze the impact of the enlargement of a common market on growth. There are two sectors in each country, including the consumption-good sector and the R&D sector. As in Romer (1990), Das (2005) and Jones (2005), we consider that the R&D sector produces new blueprints or ideas for these innovations, and hence provides the engine of growth. Assume that the talent's distribution of workers is the uniform distribution. We prove that, for the backward country, the enlargement will stimulate economic growth. In addition, for the initial integrated-economy, whether the enlargement can speed up economic growth or not depends on the average talent level of the new member country.

The remainder of this paper is organized as follow. Section 2 establishes the equilibrium growth model with heterogeneous labor, and solves for the equilibrium growth rate. Section 3 considers that the impacts of the enlargement on growth. Section 4 concludes the paper.

## The model

### Static features

Consider that the economy comprises three small open countries, countries A, M and B, each with a continuum of workers. Let $L^j$ be the measure of labor forces for country $j$ ( $j \in \{A, M, B\}$ ).Every worker's talent n is heterogeneous and perfectly observable, both to himself and to all potential employers. Hence, the talent n could represent a worker's endowment and years of schooling. Assume that the talent's distribution is the uniform distribution and has probability density function $\phi^j(n)$ for country j as shown below:

$$\phi^j(n) = \begin{cases} \dfrac{1}{b^j}, & if \ \ n \in [n^j_{\min}, n^j_{\max}], \\ 0\,, & otherwise, \end{cases}$$

Where

$$n^j_{\min} = \overline{n}^j - \frac{b^j}{2}, \ n^j_{\max} = \overline{n}^j + \frac{b^j}{2}$$

The variable $b^j$ represents the diversity of talent. The larger the variable $b^j$ is, the more diverse the distribution of talent will be. We assume that $n^j_{\min}$ and $n^j_{\max}$ are the minimum and maximum talent levels respectively and $\overline{n}^j$ is the average talent level for country j.

Each country has two sectors: a consumption-good sector and an R&D sector. Suppose that those countries are similar in their production technologies referring to the supermodular and submodular technologies, as derived by Milgrom and Roberts (1990), Kremer (1993) and Grossman and Maggi (2000). The production process involves two tasks including task x and task v in each sector. The tasks are indivisible and each task is performed by exactly one worker. In the consumption-good sector which we denote the C sector, a pair of workers performs complementary tasks. Let $\eta^j F^j_C(n_x, n_v)$ be the supermodular production function for sector C of country j when the first task (task x) is performed by a worker with talent $n_x$ and the second (task v) by a worker with talent $n_v$. For simplicity, we assume that the complementarity is extreme. Hence, the production function of sector C could be specified as:

$$\eta^j F^j_C(n_x, n_v) = \eta^j \min\{n_x, n_v\}$$

The R&D sector produces the new blueprints $\dot{\eta}^j$ (the time derivative of $\eta^j$), which accelerates technology improvement for producing the consumption-good. As in Romer (1990) and Das (2005), the level of existing technology or the stock of blueprints has a positive influence on the output of R&D sector. However, in contrast to sector C, in the R&D sector which we denote the S sector, the talent of the superior worker fully dominates the effective output and the workers toil on substitutable tasks. Let $\eta^j F_S^j(n_x, n_v)$ be the submodular production function for sector S of country j. For simplicity, we also assume that the substitutability is extreme. Thus, the production function of sector S could be specified as:

$$\eta^j F_S^j(n_x, n_v) = \eta^j \max\{n_x, n_v\}$$

Grossman and Maggi (2000) prove that in equilibrium the C sector employs the workers with similar abilities i.e. "skill-clustering" and the S sector attracts the most-talented and least-talented workers i.e. "cross-matching". We define that the variable $\hat{n}^j$ represents the least-talented worker and $m^j(\hat{n}^j) = 2\overline{n}^j - \hat{n}^j$ represents the most-talented worker in the C sector for country j. Consequently, the level of output per capital of good C (denoted by $y_C^j$) is

$$y_C^j = \frac{Y_C^j}{L^j} = \int_{\hat{n}^j}^{m^j(\hat{n}^j)} \eta^j F_C^j(n,n)\phi^j(n)dn = \frac{\eta^j \overline{n}^j}{b^j}(\overline{n}^j - \hat{n}^j) \quad (1)$$

where the variable $Y_C^j$ represents the total output of good C. As in equation (3) of Das (2005), we assume that the level of output per capital of good S must be equal to $\dot{\eta}^j$.[1] Therefore, the level of output per capital of good S (denoted by $y_S^j$) is

$$y_S^j = \frac{Y_S^j}{L^j} = \dot{\eta}^j = \int_{n_{\min}^j}^{\hat{n}^j} \eta^j F_S^j[n, m^j(n)]\phi^j(n)dn$$
$$= \frac{\eta^j}{2b^j}\left(\frac{b^j}{2} - \overline{n}^j + \hat{n}^j\right)\left(\frac{b^j}{2} + 3\overline{n}^j - \hat{n}^j\right) \quad (2)$$

[1] The main purpose is to eliminate the 'scale effects' meaning that larger economies should grow faster, as discussed in Young (1998).

where the variable $Y_S^j$ represents the total output of good S.

The production possibility frontier of country j is strictly concave and its marginal rate of transformation ($MRT^j$) can be calculated as following:

$$MRT^j = -\frac{\partial y_S^j}{\partial y_C^j} = -\frac{\partial y_S^j / \partial \hat{n}^j}{\partial y_C^j / \partial \hat{n}^j} = 2(1 - \frac{\hat{n}^j}{2\bar{n}^j}) \qquad (3)$$

Assume that preferences in the countries A, M and B are identical and homothetic. Therefore, we could characterize a competitive, free-trade equilibrium as follows:

$$p = MRT^j = 2(1 - \frac{\hat{n}^j}{2\bar{n}^j}) \qquad (4)$$

That is to say, the competitive equilibrium maximizes the national output at given relative prices, p, which represents the relative price of good C. Equation (4) determines $\hat{n}^j = (2-p)\bar{n}^j$ (time-invariant) and thereby solves our model at any time. [2]

## Growth

As analysis earlier, in equilibrium $\hat{n}^j$ is independent of time. By differentiating equation (1) with respect to time, we could derive that the growth rate of consumption goods is $g^j = \dot{\eta}^j / \eta^j$. By combining equation (2) with equation (4) and eliminating the variable $\hat{n}^j$, we could find the growth rate of country *j* as follows:

$$g^j = \frac{1}{2b^j}[\frac{b^j}{2} + (1-p)\bar{n}^j][\frac{b^j}{2} + (1+p)\bar{n}^j]. \qquad (5)$$

[2] In order to purge off the corner solution in equilibrium, we have the relationship of $1 < p < 2$ and hence implies that $\hat{n}^j = (2-p)\bar{n}^j > 0$ holds.

There is no transitional dynamics. As we can see from equation (5), the factors affecting the growth rate include the diversity of talent, the world relative price and the average talent level. From equation (5), we can easily obtain:

$$\frac{\partial g^j}{\partial b^j} = \frac{1}{2}\left(\frac{\bar{n}^j}{b^j}\right)^2\left[\left(\frac{b^j}{2\bar{n}^j}\right)^2 - (1-p)(1+p)\right] > 0\,, \qquad \text{(6a)}$$

$$\frac{\partial g^j}{\partial p} = -\frac{p\bar{n}^{j^2}}{b^j} < 0\,, \qquad \text{(6b)}$$

$$\frac{\partial g^j}{\partial \bar{n}^j} = \frac{\bar{n}^j}{b^j}\left[\frac{b^j}{2\bar{n}^j} + (1-p)(1+p)\right] \gtreqless 0\,. \qquad \text{(6c)}$$

Equation (6a) indicates that a rise in the diversity of talent will stimulate the growth rate, as proven in Das (2005). Equation (6b) postulates that an increase in the relative price of good *C* will be detrimental to economic growth. The economic intuitions behind the story are that a rise in the diversity of talent or a decrease in the relative price of good *C* will lead to more output of good *S* and thereby speed up the growth rate. The ambiguous result expressed in equation (6c) can be explained through the following two aspects. First, a rise in the average talent level increases the productivity of workers, and hence raises the output of good *S*. Second, as in Grossman and Maggi (2000), a rise in the average talent level leads to less aggregate talent allocated to sector S, and thereby reduces the output of good S. It is obvious that the two aspects summarize an ambiguous response in output of good S[3]. Therefore, the impact of the average talent level on the growth rate is also ambiguous.

[3] From equation (4), we can derive the relationship of $0 < \partial\hat{n}^j / \partial\bar{n}^j = (2-p) < 1$ implying that a rise in the average talent level will lead to less aggregate talent employed in sector S.

## Enlargement of the common market and growth

In this section, we will analyze the impact of the enlargement of a common market on economic growth. For simplicity and without losing generality, suppose that the diversities of talent and the measures of labor forces in countries $A$, $M$ and $B$ are identical, i.e., $b^A=b^M=b^B=b$ and $L^A=L^M=L^B=L$.[4] In our paper, country $A$ is advanced country with the larger average talent level, country $M$ is developing country with the middle average talent level and country $B$ is backward country with the smaller average talent level. Assume that $n_{\min}^A = n_{\max}^M$ and $n_{\min}^M = n_{\max}^B$. Thus, the difference of the average talent levels of countries $A$ and $M$ or countries $M$ and $B$ is the diversity of talent, i.e., $\bar{n}^A = \bar{n} + 2b$, $\bar{n}^M = \bar{n} + b$ and $\bar{n}^B = \bar{n}$.

### The initial common market

Consider that countries $A$ and $M$ initially join together to create a common market. Namely, workers could be mobile freely between member countries. We make use of the superscript "$I$" to denote the variables after the initial integrated-economy formed by countries $A$ and $M$. Thus, $L^I$ is the measure of labor forces ($L^I = L^A + L^M$) and $\phi^I(n)$ represents the probability density function of talent. In addition, we assume that $n_{\min}^I = \bar{n}^I - (b^I/2)$ and $n_{\max}^I = \bar{n}^I + (b^I/2)$ are the minimum and maximum talent levels respectively. At the same time, $\bar{n}^I$ is the average talent level and $b^I$ denotes the diversity of talent. Therefore, under certain situation as will be considered in this paper, the $\phi^I(n)$ will also be an uniform distribution. By using the analytical method of Section 2, we could derive the $g^I$ representing the growth rate of the initial integrated-economy as shown below:

$$g^I = \frac{1}{2b^I}\left[\frac{b^I}{2} + (1-p)\bar{n}^I\right]\left[\frac{b^I}{2} + (1+p)\bar{n}^I\right] \qquad (7)$$

4 Since we have purged scale effects from the model, the effect of aggregate labor forces on the growth rate after enlargement is absent.

As assumption earlier (i.e., $n_{\min}^{A} = n_{\max}^{M}$ , $\overline{n}^{A} = \overline{n} + 2b$ and $\overline{n}^{M} = \overline{n} + b$ ), the probability density function $\phi^{I}(n)$ would be:

$$\phi^{I}(n) = \begin{cases} \dfrac{1}{2b}, & if \;\; n \in [n_{\min}^{I}, n_{\max}^{I}], \\ 0\,, & otherwise, \end{cases}$$

where

$$n_{\min}^{I} = n_{\min}^{M} = \overline{n} + \frac{b}{2},\; n_{\max}^{I} = n_{\max}^{A} = \overline{n} + \frac{5b}{2}.$$

Note that integration of countries *A* and *M* would lead to the changes of the probability density functions of talent and the average talent levels, i.e., $\phi^{I}(n) = 1/2b$ and $\overline{n}^{I} = \overline{n} + 3b/2$ . By substituting the relationships of $b^{I}$=2*b* and $\overline{n}^{I} = \overline{n} + 3b/2$ into equation (7), we could find the growth rate of the initial integrated-economy as follows:

$$g^{I} = \frac{1}{4b}[b + (1-p)(\overline{n} + \frac{3b}{2})][b + (1+p)(\overline{n} + \frac{3b}{2})] \qquad (8)$$

**Enlargement of the common market**

Now the third country *B* will be integrated into the initial common market formed by countries *A* and *M*. Namely, country *B* is the new member country. We make use of the superscript "*En*" to denote the variables after the enlargement of common market. Thus, $L^{En}$ is the measure of labor forces ($L^{En} = L^{A} + L^{M} + L^{B}$) and $\phi^{En}(n)$ represents the probability density function of talent. In addition, we assume that $n_{\min}^{En} = \overline{n}^{En} - (b^{En}/2)$ and $n_{\max}^{En} = \overline{n}^{En} + (b^{En}/2)$ are the minimum and maximum talent levels respectively. At the same time, $\overline{n}^{En}$ is the average talent level and $b^{En}$ denotes the diversity of talent. Therefore, under certain situation as will be considered in this paper, the $\phi^{En}(n)$ will also is a uniform distribution.

Using the analytical method of Section 2 could derive the $g^{En}$ representing the growth rate after enlargement as shown below:

$$g^{En} = \frac{1}{2b^{En}}[\frac{b^{En}}{2} + (1-p)\overline{n}^{En}][\frac{b^{En}}{2} + (1+p)\overline{n}^{En}] \qquad (9)$$

As assumption earlier (i.e., $n^{A}_{\min} = n^{M}_{\max}$ , $n^{M}_{\min} = n^{B}_{\max}$ , $\overline{n}^{A} = \overline{n} + 2b$, $\overline{n}^{M} = \overline{n} + b$ and $\overline{n}^{B} = \overline{n}$), the probability density function $\phi^{En}(n)$ would be:

$$\phi^{En}(n) = \begin{cases} \dfrac{1}{3b}, & if \ \ n \in [n^{En}_{\min}, n^{En}_{\max}], \\ 0, & otherwise, \end{cases}$$

where

$$n^{En}_{\min} = n^{B}_{\min} = \overline{n} - \frac{b}{2}, \ n^{En}_{\max} = n^{A}_{\max} = \overline{n} + \frac{5b}{2}.$$

Note that the enlargement would also lead to the changes of the probability density functions of talent and the average talent levels, i.e., $\phi^{En}(n) = 1/3b$ and $\overline{n}^{En} = \overline{n} + b$. Again, substituting the relationships of $b^{B}$=$b$ and $\overline{n}^{B} = \overline{n}$ into equation (5) could obtain the growth rate of country *B* (the new member country) before the enlargement as follows:

$$g^{B} = \frac{1}{2b}[\frac{b}{2} + (1-p)\overline{n}][\frac{b}{2} + (1+p)\overline{n}] \qquad (10)$$

By substituting the relationships of $b^{En}$=3$b$ and $\overline{n}^{En} = \overline{n} + b$ into equation (9), we could find the growth rate after the enlargement as follows:

$$g^{En} = \frac{1}{6b}[\frac{3b}{2} + (1-p)(\overline{n} + b)][\frac{3b}{2} + (1+p)(\overline{n} + b)] \qquad (11)$$

It will not be difficult to analyze the impacts of the enlargement of a common market on economic growth. First, we consider the growth effect of the enlargement on the new member country. From equations (10) and (11), we could derive the difference of growth rates for country *B* before and after the enlargement as follows:

$$g^{En} - g^{B} = \frac{1}{6b}[2(\overline{n} - \frac{b}{2})^2 (p^2 - 1) + \frac{3b^2}{2}(4 - p^2)] > 0 \qquad (12)$$

Equation (12) claims that the impact of the enlargement on the growth rate for backward country (country *B*) is positive. The economic intuition is that both rises in the diversity of talent and the average talent level after the enlargement, from the backward country's point of view, will lead to more output of good *S* and hence stimulate growth. Therefore, this result is formalized in the following proposition.

**Proposition 1.** *The enlargement of common market will be conducive to economic growth for backward country*

Next, we will explore the growth effect of the enlargement on the initial common market formed by countries *A* and *M*. From equations (8) and (11), the difference of growth rates for the initial common market before and after the enlargement is as follows:

$$g^{En} - g^{I} = \frac{b}{8[\Omega^2(b,\overline{n}) - 1]}[p + \Omega(b,\overline{n})][p - \Omega(b,\overline{n})] \qquad (13a)$$

where

$$\Omega(b,\overline{n}) = (1 + \frac{6b^2}{4\overline{n}^2 + 20b\overline{n} + 19b^2})^{0.5},\ 1 < \Omega(b,\overline{n}) < 2 \qquad (13b)$$

Therefore, we get

$$g^{En} - g^{I} \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} 0 \ \text{, if} \left\{ \begin{array}{l} \Omega(b,\overline{n}) < p < 2 \\ p = \Omega(b,\overline{n}) \\ 1 < p < \Omega(b,\overline{n}) \end{array} \right. \qquad (13c)$$

Equation (13c) indicates that whether the growth rate for the initial common market after the enlargement rises or not depends on the world price p, which in turn could be described in Figure 1. As we can see from equation (13b), the factors affecting the critical point $\Omega(b,\overline{n})$ include the diversity of talent and the average talent level. However, under certain situation as has been considered in this section, the only difference of countries A, M and B is the average talent level (i.e., bA=bM=bB=b, $\overline{n}^A = \overline{n} + 2b$, $\overline{n}^M = \overline{n} + b$ and $\overline{n}^B = \overline{n}$). Therefore, we will analyze the impact of the average talent level on the critical point. From equation (13b), we could find that the higher value of $\overline{n}$ would lead to a lower critical value of $\Omega(b,\overline{n})$.5 As we can observe from Figure 1, when the average talent level $\overline{n}$ rises, the critical point $\Omega(b,\overline{n})$ will shift left. That is to say, the higher the average talent level for the new member country (country B) is, the more possible an increase in the growth rate for the initial common market after the enlargement will be. This feature is summarized as Proposition 2:

**Proposition 2.** *The higher the average talent level for the new member country, the more likely it is that the enlargement will stimulate economic growth for the initial integrated-economy.*

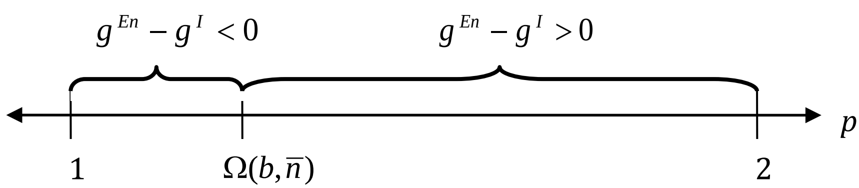


**Figure 1**: Terms of trade and growth rate

5 $\partial\Omega(b,\overline{n}) / \partial\overline{n} = -\{(5b + 2\overline{n})[\Omega^2(b,\overline{n}) - 1]^2\} / [3b^2\Omega(b,\overline{n})] < 0$.

## Conclusions

By using a three-country, two-sector equilibrium growth model with heterogeneous labor, we have analyzed the effects of the enlargement of a common market on the member countries' growth. We prove that, for the backward country, the enlargement will stimulate economic growth. In addition, for the initial integrated-economy, we demonstrate that the higher the average talent level of the new member country is, the more likely it is that the enlargement can speed up economic growth. Our results have sharp contrasts to the one by Walz (1998) who shows that the enlargement might lead to a reduction in growth due to migration for unskilled labor or emigration for skilled labor.

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