Using Linear Programming in order to Optimize the Allocation of Resources for Investment

Authors: **Rodica Gherghina,** The Bucharest University of Economic Studies, Faculty of Finance, Insurance, Banking and Stock Exchange,Finance Department, Bucharest, Romania rodica.gherghina@fin.ase.ro

Ioana Duca, Titu Maiorescu University, Faculty of Economic Sciences, Finance, Banking and Business Administration Department, Bucharest, Romania, ioana.duca@utm.ro

The authors' scientific approach highlights the method to allocate resources for investments to modernize two laboratories within a school, using linear programming. The study aims to achieve maximum profit by upgrading the two laboratories. We made recommendations with respect to the optimum solution. However, using optimization conception is only possible if the conditions for achieving the relevant decisions are correctly formulated and the efficiency criterion is known. Using linear programming method recommendations can be developed related to economically optimal decisions regarding especially the investment resource allocation.

Keywords: *optimization, linear programming, investments*

Jel Classification: *C0,C1, C4,C6*

Introduction

The selection of the optimum variants regarding the project, the optimal use of material, human and time resources can be solved resorting to mathematical methods of optimization. These include the linear programming method. However, the use of the optimization concept is only possible if the conditions for achieving the relevant decisions are correctly formulated and the efficiency criterion is known.

Nevertheless, the results obtained by solving the optimization problems that are otherwise mathematically correct, are effective means of substantiating the decision, which means initially must be reviewed and then assessed until the time of decision

In this context, optimization methods are an useful scientific instrument to approach, settle and adopt solutions, requiring several steps in optimization problems modeling and solving to be covered, say experts in economics Despa, Avrigeanu, Zirra, Munteanu (2005, pp. 104-105), such as:

• "selecting the economic efficiency criterion of the optimization objective function;

• identifying those factors which influence the objective function and which of these factors can be controlled or adjusted, in order their number eventually to be as small as possible;

• establishing restrictions, minimum, respectively maximum levels of resources used in implementing the investment objectives;

• establishing the mathematical model of the objective function for the optimization problem;

• effective implementation of the mathematical model, determining its solution and also finding the optimal variant;

• verifying the stability of the solution obtained against the assumptions adopted;

• formulating the recommendation on the optimal variant and indicating the advantage of this variant as compared with the others."

Literature review

By using linear programming method, the specialists economics, Staicu (2000), states that recommendations can be developed on optimal economic decisions regarding especially the resource allocation for investment. According to many specialists: Johnston(1991);Gujarati (1995);Pecican(1996);Capanu,Wagner,Secăreanu,(1997);Cistelecan(2002);Pârvu,Andreica(2000);Pârvu(2004);Vasilescu,Gheorghe,Cicea,Dobrea(2004);Despa,Zirra,Avrigeanu,Munteanu(2005); Taşnadi(2005),the optimal variant of an investment project is considered the one that ensures a maximum return or allows decisions to be implemented with minimum efforts.

Description and solving the model

We present the following example linear programming model (adaptation after Staicu, 2000, pp.66-69):

We have the amount of 112,5 mii lei for the modernization of two laboratories in a high school. Based on the data presented in table 1, and with a view an as high as possible profit (benefit) to being obtained the distribution of the specified amount is required to be achieved.

Table 1: Investment and profits earned by upgrading laboratories

	Laboratories	Information	Chemistry
Investment resources	X	Y	112,5 mii lei
Investment	1,07	2,12	225 mii lei
Benefit	0,17	0,19	maxim

In order to build the mathematical model we will initially calculate the profit for an invested amount of Lei 1 so that we can relate to the variable of the problem.

1. For Information Laboratory:
2. For Chemistry Laboratory:

The mathematical model that we get is:

$$\max F(x,y) = 0{,}1819 * x + 0{,}4028 * y$$

$$x + y \leq 112{,}5$$

$$1{,}07 * x + 2{,}12 * y \leq 225$$

$$\text{where: } x \geq 0;\ 0;\ y \geq 0$$

The solving method will be the graphic method, because we have only two unknown variables. We initially proceed to transform the inequalities in equalities so that the corresponding straight lines to be drawn on the graph:

$$d_1 : x + y = 112{,}5$$

$$d_2 : 1{,}07 * x + 2{,}12 * y = 225$$

Since we have Ist degree equations, these can be solved by successive cancellation of the variables, for each of them:

$$d_1 : \begin{cases} x = 0;\ y = 112{,}5 \\ y = 0;\ x = 112{,}5 \end{cases}$$

$$d_2 : \begin{cases} x = 0;\ y = 225/2{,}12 = 106{,}13 \\ y = 0;\ x = 225/1{,}07 = 210{,}28 \end{cases}$$

Then, using the points whose coordinates we obtained (0; 112,5) and (112,5; 0) we drew the straight line d1, and with coordinates (0; 106,13) and (210,28; 0) we drew the straight line d2.

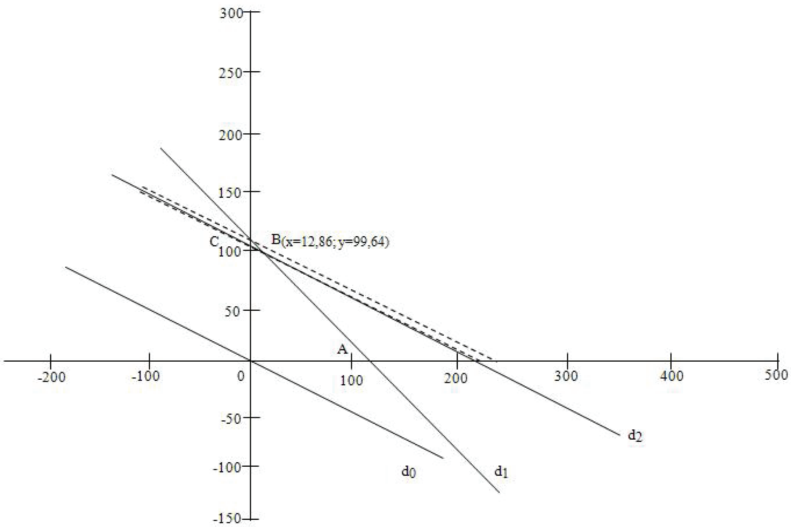

Figure 1: Graphic representation made by the authors based on the results

Given the imposed restrictions we determined the solution contained among A, B, C, but we consider sign of inequality ≤. Thus, we remark that the solutions are both on the straight lines but also under them.

Next, to determine the coordinates of point B which lies at the crossing of lines d1 and d2, we have developed a system of equations (with the two equalities that represent the straight lines), which we solve by the method of reduction.

$$\begin{cases} x + y = 112{,}5 & -1{,}07 \\ 1{,}07 * x + 2{,}12 * y = 225 \end{cases}$$

$$\begin{cases} -1{,}07 * x - 1{,}07 * y = -120{,}38 \\ 1{,}07 * x + 2{,}12 * y = 225 \end{cases}$$

$$=> 1{,}05y = 104{,}62 \;=>\; y = 104{,}62/1{,}05 = 99{,}64$$

$$=> x = 112{,}5 - 99{,}64 = 12{,}86$$

To identify the optimal solution (from among the large number of possible solutions) we graphically represent the objective function, the straight line do, which is seen as passing through the origin of coordinates. Thus, parallels to this straight line are drawn through the points with solutions, and of course the best solution is the point farthest from the origin, only if the objective function is a maximum function, respectively, the nearest point if the objective function is a minimum function.

By equalizing the objective function with 0 we obtain:

$$d_0 = 0{,}1819 * x + 0{,}4028 * y = 0$$

And we give the variables values greater than 0, close to the measurement unit of the graph.

$$\begin{cases} x = 100;\ y = -18{,}19/0{,}4028 = -45{,}15 \\ y = 100;\ x = -40{,}28/0{,}1819 = -221{,}44 \end{cases}$$

We note that in the point C (0; 106,13) we find the optimal solution, namely x = 0, y = 106,13 mii lei. And the value of the objective function, for the optimal solution is:

$$\max F(x,y) = 0{,}1819 * 0 + 0{,}40$$

To check the solution we introduce in the objective function the coordinates of all the points with possible solutions.

$$F_A = 0{,}1819 * 112{,}5 + 0{,}4028 * 0 = 20{,}46 \text{ mii lei}$$

$$F_B = 0{,}1819 * 12{,}86 + 0{,}4028 * 99{,}64 = 42{,}47 \text{ mii lei}$$

$$F_C = 0{,}1819 * 0 + 0{,}4028 * 106{,}13 = 42{,}75 \text{ mii lei}$$

In this context, the parameter values in the three points will be:

Investment resources		Investment	Benefit
A	112,5	224,999	20,46
B	112,5	224,997	42,47
C	106,13	224,995	42,75

The authors made the calculation by introducing the coordinates of the points in the relevant equations.

For example, in the cost equation:

$$A : 1{,}07 * 210{,}28 + 2{,}12 * 0 = 224{,}999$$

$$B : 1{,}07 * 12{,}86 + 2{,}12 * 99{,}64 = 224{,}997$$

$$C : 1{,}07 * 0 + 2.12 * 106{,}13 = 224{,}995$$

We see that if the entire amount of funds for investment was not used, the disadvantage of the solution appears in point C.

Conclusions

About the optimal solution the following recommendations can be made. For example, it is not recommended the resources to be allocated for the modernization of one laboratory only. In this way, the solution from point B can be used, i.e. the distribution of resources on the two laboratories, if we have in view that the obtained profit is close to the profit obtained in variant C and the chances to achieve it are higher, as it satisfies the modernization needs for the two laboratories.

References

[1] Capanu,I., Wagner, P., Secăreanu, C., Statistică Macroeconomică, Editura Economică, București, 1997

[2] Cistelecan, L., Economia eficiența și finanțarea investițiilor, Editura Economică, București, 2002

[3] Despa, R., Zirra, D., Avrigeanu, A., Munteanu, A., C., Eficiența investițiilor, Editura Universitară,București, 2005

[4] Gherghina, R., Optimizarea și eficientizarea cheltuielilor pentru educație, Editura

Universitară, București, 2009
[5] Gujarati, D., N., Basic Econometrics, Mc. Grow Hill Inc N. Y., 1995
[6] Johnston, J., Econometric Methods, Mc. Grow Hill, N.Y., 1991
[7] Pârvu, D., Andreica, M., Eficiența și finanțarea investițiilor, Editura Cibernetică MC, București, 2003
[8] Pârvu, D., Eficiența investițiilor, Editura Lumina Lex, București, 2004
[9] Pecican, E., S., Econometrie, Editura All, București, 1996
[10] Românu, I., Econometrie cu aplicații la eficiența investițiilor, Editura Științifică și Enciclopedică, București, 1975
[11] Românu, I., Vasilescu, I., Eficiența economică a investițiilor și a capitalului fix, Editura Didactică și Pedagogică, București, 1993
[12] Staicu, F.(coordonator), Eficiența economică a investițiilor, Editura Didactică și Pedagogică, București, 1995
[13] Staicu, F., Eficiența economică a investițiilor, Editura Secorex, București, 2000
[14] Tașnadi, A., Econometrie, Editura ASE, București, 2005
[15] Vasilescu, I., Gheorghe A., Cicea, C., Dobrea, C., Eficiența și evaluarea investițiilor, Editura EfiCon Press, București, 2004