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Optimal Work Effort and Monitoring Cost

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Using a simple job market equilibrium model we study the relationship between work effort and monitoring by firms. Some other determinants of work effort investigated include the educational level of the worker, the minimum or start-up salary as well as the economic conjuncture. As common logic dictates, optimal work effort increases with the amount of monitoring done by the employer. Quite contrary to common logic, though, we find that at the optimum employers observe and control good workers much more stringently and meticulously than poor workers. This is because under profit maximization most of the employer's profit and surplus result from good workers and he risks losing a large amount of profit by not observing those. Managers monitor strictly more productive workers, fast learners and those starting at a higher autonomous level of monitoring, as those contribute more substantially to the firm's profit.

Keywords: *labor, productivity of labor, learning curve, learning-by-doing, work effort, monitoring, minimum wage*

Introduction

A large part of the literature on labor economics is dedicated to job search. Some of the first to study the problem of search effort in the conditions of costly information and high uncertainty were Stigler (1961, 1962) and Alchian and Allen (1964). McCall (1970) put this essential problem in a mathematical framework with relevance to reservation wage, i.e., the lowest wage the worker is willing to accept where the reservation wage may change over time and will differ for two jobs of different characteristics; that is, there will be compensating wage differentials between different types of jobs. McCall's basic model serves as a basis for later works among which those of Mortensen (1977), Burdett (1978), Pissarides (1994), and Van den Berg (1990). These authors analyze the behavior of unemployment (Pissarides 1994), wage determination (Burdett (1978)), job duration, job turnover (Van den Berg 1992), quit rates (Burdett 1978), and unemployment insurance and employment protection (Pissarides 1994). Kahn (1978) investigates the relationship between search time and resulting wage as well as the duration span of unemployment.

Becker (1967) develops the first model on optimal human capital investment. He studies how ability and family wealth interact to determine the distribution of lifetime earnings. Human capital investment depends on family incomes since families pay for the post-secondary schooling required to gain additional skills. In this sense, wealthy agents can obtain better education that promises higher future earnings. Becker defines this difference in financing education as "unequal opportunity." Since poor individuals face higher opportunity cost of financing their education, they are likely to remain less educated even when they have the same personal abilities as rich individuals.

Learning models in the theory of earnings distributions (Faber and Gibbons (1996), Jovanovic (1979)) state that without information about their relative talents for different types of jobs, workers cannot make choices that maximize earnings. Workers gain from work experience by receiving information about their skills that can boost future earnings. Low-income agents accept low-paid jobs and tend to gain less valuable experience at the workplace. This decreases the quality of their sorting process and ultimately lowers their lifetime earnings. High-income agents employed in highly-skilled positions gain more from work experience which improves their opportunities of finding even better jobs.

Part of the literature is dedicated to minimum and equilibrium wage. Some studies go beyond the traditional supply and demand analysis of the labor market which ignores price effects. Fields (1994) proposes a more complicated analysis of minimum wage where there are two sectors of the economy, one involving workers exempt from the minimum wage. Gillespie (2007) argues that if the demand for the good produced by the firm is very inelastic, management can offset the negative effects of the higher wage floor by raising prices. Since demand is highly inelastic, the firm would not incur losses and would not have to fire employees. Some authors (Katz and Krueger (1992), Card and Krueger (1995)) view the labor market as monopsonistic. Thus employers have greater market power in setting wages than employees.

This monopsony could be the result of employer collusion or natural factors such as segmented markets, search costs, information costs, imperfect mobility, etc. As a result of such market failure employees are paid less than their marginal value. An appropriately set minimum wage could increase both wages and unemployment, with the optimal level being equal to the marginal productivity of labor.

Much of the economic literature discusses monitoring in relation to the shirking problem. Shirking occurs on the job and represents a form of opportunism on the part of the worker who does not abide by his work contract. Therefore, the employer must exert control over the actions of the worker so that the amount of his effort is maximized and he gives the maximum marginal product of his labor. It may not always be in the interest of the worker to fulfill his obligations fully – he will exert effort only to the point where it pays him to do so. Alchian and Demsetz (1972) study the need for monitoring team effort, i.e., when the free-rider problem arises and the post-contractual opportunism of one team member affects the welfare of the other members. To new institutional economists the firm exists to reduce shirking by lowering monitoring cost and directing the organization of jointly cooperating inputs (Williamson (1975), Klein, Crawford, and Alchian (1978), Barzel (1984)). Shapiro and Stiglitz (1987) put shirking and the absence of work effort in a specific model which presents the threat of dismissal as a corrective measure for labor.

We study work effort with its various determinants, the most important one being the monitoring cost and control efforts of the employer. Some other determinants include the educational level of the worker, the minimum or start-up salary as well as the conjuncture, i.e., whether the worker exerts the effort in recessionary conditions or an economic boom. We perceive work effort as the skillfulness of labor, not as its marginal product. In fact using optimization techniques we find that work effort and the marginal product of labor are just two opposites, where a higher marginal productivity of labor reduces the optimal effort of the worker. Thus, work effort can be defined as the amount of work, effort and hardships the worker endures on the job. At the same time, the marginal product of each worker can be equated to the outcome of his activities. While the latter is equivalent to product and is product-oriented, the former is process-oriented. Since most productive and skillful workers are smooth and efficient, much of the time they end up exerting less effort than less skilled, less talented ones. Thus more educated people would spend less time fulfilling a task. At the same time, people with no talent exert strenuous efforts but rarely achieve good results. In relation to worker talent is employer's monitoring cost. By means of optimization we find that optimal work effort depends directly on the amount of monitoring done by the employer. Much to our surprise and contrary to the common belief that more productive workers need less monitoring and control we find that at the optimum employers would observe and control good workers much more stringently and meticulously than poor workers. This is because under profit maximization most of the employer's profit and surplus result from good workers and he risks losing a large amount of profit by not observing those.

The paper is organized as follows: Part 1 is a literature review and a discussion of the rationale for the paper. Part 2 gives a simple job market equilibrium model with educational level, minimum wage and monitoring cost as determinants of work effort. Part 3 discusses the relationship between work effort and monitoring cost in relation to economic conjuncture. Part 4 reveals the effect of monitoring on work effort under the conditions of profit maximization. Part 5 extends these results to the unconstrained case of profit maximization where two individuals are employed, one more productive than the other. The paper ends with conclusions.

A simple job market equilibrium model

Assuming a simple equilibrium model on the job market, we equate supply of labor to exogenous demand for it. Supply S is positively related to wage w and effort e on the job. Thus, the more efforts workers exert, the greater the overall supply of labor. Greater monitoring m by the employer also increases the overall supply of labor, therefore, $S_m > 0$

$$S(w,e,m) = D_o \qquad S_w > 0 \qquad S_e > 0 \qquad S_m > 0$$

On the other hand, wage is determined by the minimum wage or start-up pay level for the worker w_o but it also grows with the educational level s_o, assumed here to be exogenous, as well as the effort e of the individual worker. Thus harder working individuals are rewarded with an increase in the overall wage.

$$w = w_o + g(e,s_o) \qquad g_e > 0 \qquad g_{s_o} > 0$$

Finally, work effort depends on its initial, autonomous level e_o and on the amount of monitoring m exercised by the employer. Starting from a higher initial level of effort hard working individuals tend to exert a higher overall work effort. Therefore,

$$e = e_o + h(m) \qquad\qquad h_m > 0$$

Rearranging the equations in the form of implicit functions,

$$S(w,e,m) - D_o = 0$$
$$w - w_o - g(e,s_o) = 0$$
$$e - e_o - h(m) = 0$$

we write off the following matrix equation differentiating the endogenous variables with respect to the educational level and applying the implicit-function theorem.

$$\begin{bmatrix} S_w & S_e & S_m \\ 1 & -g_e & 0 \\ 0 & 1 & -h_m \end{bmatrix} \begin{bmatrix} \dfrac{\partial w}{\partial s_o} \\ \dfrac{\partial e}{\partial s_o} \\ \dfrac{\partial m}{\partial s_o} \end{bmatrix} = \begin{bmatrix} 0 \\ g_{s_o} \\ 0 \end{bmatrix} \qquad |J| = S_w g_e h_m + S_e h_m + S_m > 0$$

where the Jacobian is definitely positive. By matrix inversion,

$$C = \begin{bmatrix} g_e h_m & h_m & 1 \\ h_m S_e + S_m & -h_m S_w & -S_w \\ g_e S_m & S_m & -g_e S_w - S_e \end{bmatrix} \quad C' = \begin{bmatrix} g_e h_m & h_m S_e + S_m & g_e S_m \\ h_m & -h_m S_w & S_m \\ 1 & -S_w & -g_e S_w - S_e \end{bmatrix}$$

$$J^{-1} = \frac{1}{|J|} \begin{bmatrix} g_e h_m & h_m S_e + S_m & g_e S_m \\ h_m & -h_m S_w & S_m \\ 1 & -S_w & -g_e S_w - S_e \end{bmatrix} = \frac{1}{(S_w g_e h_m + S_e h_m + S_m)} \begin{bmatrix} g_e h_m & h_m S_e + S_m & g_e S_m \\ h_m & -h_m S_w & S_m \\ 1 & -S_w & -g_e S_w - S_e \end{bmatrix}$$

$$\begin{bmatrix} \frac{\partial w}{\partial s_o} \\ \frac{\partial e}{\partial s_o} \\ \frac{\partial m}{\partial s_o} \end{bmatrix} = \frac{1}{(S_w g_e h_m + S_e h_m + S_m)} \begin{bmatrix} g_e h_m & h_m S_e + S_m & g_e S_m \\ h_m & -h_m S_w & S_m \\ 1 & -S_w & -g_e S_w - S_e \end{bmatrix} \begin{bmatrix} 0 \\ g_{s_o} \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{g_{s_o}(h_m S_e + S_m)}{S_w g_e h_m + S_e h_m + S_m} \\ -\frac{g_{s_o} h_m S_w}{S_w g_e h_m + S_e h_m + S_m} \\ -\frac{g_{s_o} S_w}{S_w g_e h_m + S_e h_m + S_m} \end{bmatrix}$$

More specifically, with respect to educational level we obtain several comparative-static effects on the endogenous variables. As can be expected, the effect of education on wage is favorable. More educated people are likely to receive a higher total wage, where,

$$\frac{\partial w}{\partial s_o} = \frac{g_{s_o}(h_m S_e + S_m)}{S_w g_e h_m + S_e h_m + S_m} > 0$$

At the same time, more educated individuals need not exert much effort on the job. Contrary to the common belief that educated people try harder, we obtain that they actually need to work less, as they perhaps achieve a given result with less effort. Uneducated people have to try a lot harder on the job to achieve the same results as skillful, educated workers. Hence, education and work effort are adversely related.

$$\frac{\partial e}{\partial s_o} = -\frac{g_{s_o} h_m S_w}{S_w g_e h_m + S_e h_m + S_m} < 0 \qquad \frac{\partial m}{\partial s_o} = -\frac{g_{s_o} S_w}{S_w g_e h_m + S_e h_m + S_m} < 0$$

Higher education requires less monitoring. This could be explained with the result obtained previously, that educated people achieve the goals or perform the tasks much more swiftly and smoothly than others. But this may also be due to the fact that the employer trusts the credentials and performance of skillful, well trained workers much more than unskilled ones. Education serves as a signal of quality and consciousness and allows the employer to sift out the better workers. Therefore, education also brings about a higher degree of trust on the part of the employer. With respect to initial wage w_o :

$$\begin{bmatrix} \frac{\partial w}{\partial w_o} \\ \frac{\partial e}{\partial w_o} \\ \frac{\partial m}{\partial w_o} \end{bmatrix} = \frac{1}{(S_w g_e h_m + S_e h_m + S_m)} \begin{bmatrix} g_e h_m & h_m S_e + S_m & g_e S_m \\ h_m & -h_m S_w & S_m \\ 1 & -S_w & -g_e S_w - S_e \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{h_m S_e + S_m}{S_w g_e h_m + S_e h_m + S_m} \\ -\frac{h_m S_w}{S_w g_e h_m + S_e h_m + S_m} \\ -\frac{S_w}{S_w g_e h_m + S_e h_m + S_m} \end{bmatrix}$$

we obtain the following comparative-static effects:

$$\frac{\partial w}{\partial w_o} = \frac{h_m S_e + S_m}{S_w g_e h_m + S_e h_m + S_m} > 0$$

As can be expected, a higher start-up salary leads to a higher overall gross salary. Thus individuals with better credentials, education, work experience and reputation starting at a higher wage are likely to receive a higher total wage.

$$\frac{\partial e}{\partial w_o} = -\frac{h_m S_w}{S_w g_e h_m + S_e h_m + S_m} < 0$$

At the same time, a higher initial salary reduces the overall effort of the worker. This may be because a higher initial wage unrelated to work effort may be demotivating workers to contribute to the production process. But it may also reflect the fact that more skillful workers starting at a higher start-up salary end up exerting less effort as they can perform the task more efficiently and skillfully. Low-wage earners with less education have to try a lot harder to achieve the same results.

$$\frac{\partial m}{\partial w_o} = -\frac{S_w}{S_w g_e h_m + S_e h_m + S_m} < 0$$

A higher initial wage decreases the monitoring cost to the employer. This may again be related to the talent of the worker, with higher talent or skills rewarded by a higher initial wage and monitored less stringently by the manager due to the “trust” relationship with the skillful worker. But this may also be due to the high opportunity cost of losing a well-paid job for the more efficient worker. This high initial wage and, consequently opportunity cost, for him ensures good performance and strengthens the “trust” relationship between him and the employer. With respect to initial effort e_o we solve

$$\begin{bmatrix} \frac{\partial w}{\partial e_o} \\ \frac{\partial e}{\partial e_o} \\ \frac{\partial m}{\partial e_o} \end{bmatrix} = \frac{1}{(S_w g_e h_m + S_e h_m + S_m)} \begin{bmatrix} g_e h_m & h_m S_e + S_m & g_e S_m \\ h_m & -h_m S_w & S_m \\ 1 & -S_w & -g_e S_w - S_e \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{g_e S_m}{S_w g_e h_m + S_e h_m + S_m} \\ \frac{S_m}{S_w g_e h_m + S_e h_m + S_m} \\ -\frac{g_e S_w}{S_w g_e h_m + S_e h_m + S_m} \end{bmatrix}$$

A higher autonomous level of work effort e_o unrelated to the process of monitoring has a positive effect on gross salary. This initial work effort is what the worker starts with and is indicative of his reputation and qualities as worker. Thus, salary is positively related to the reputation of the worker and his credentials when starting on the job, as they speak of his consciousness and work habits.

$$\frac{\partial w}{\partial e_o} = \frac{g_e S_m}{S_w g_e h_m + S_e h_m + S_m} > 0$$

As could be expected, a higher autonomous effort, unrelated to monitoring, contributes to a greater total work effort. Thus, the more conscientious and hard working individuals starting at a higher autonomous effort tend to exert a greater amount of overall effort, irrespective of the degree of monitoring and control.

$$\frac{\partial e}{\partial e_o} = \frac{S_m}{S_w g_e h_m + S_e h_m + S_m} > 0$$

Since the more productive and serious worker starts at a higher level of effort e_o , he is less likely to be subject to monitoring and scrutiny. Thus individuals starting as good workers would be monitored less strictly than poor workers. The employer's perception of the worker determines the degree of control over his work, with workers hired as more productive from the very outset being less observed than those hired as average or poor.

$$\frac{\partial m}{\partial e_o} = -\frac{g_e S_w}{S_w g_e h_m + S_e h_m + S_m} < 0$$

As a control variable, exogenous demand   has the following effect on the three endogenous variables:

$$\begin{bmatrix} \frac{\partial w}{\partial D_o} \\ \frac{\partial e}{\partial D_o} \\ \frac{\partial m}{\partial D_o} \end{bmatrix} = \frac{1}{(S_w g_e h_m + S_e h_m + S_m)} \begin{bmatrix} g_e h_m & h_m S_e + S_m & g_e S_m \\ h_m & -h_m S_w & S_m \\ 1 & -S_w & -g_e S_w - S_e \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{g_e h_m}{S_w g_e h_m + S_e h_m + S_m} \\ \frac{h_m}{S_w g_e h_m + S_e h_m + S_m} \\ \frac{1}{S_w g_e h_m + S_e h_m + S_m} \end{bmatrix}$$

where greater market demand clearly has a positive effect on equilibrium wage such that

$$\frac{\partial w}{\partial D_o} = \frac{g_e h_m}{S_w g_e h_m + S_e h_m + S_m} > 0$$

Furthermore, it could easily be checked that demand has a positive effect on the overall effort exerted due to the increase in the total amount of labor used and, as a result, on the total monitoring of firms.

Monitoring work effort in recessionary conditions

We have so far revealed the monitoring done by the firm as the dependent variable. More specifically, we demonstrated how monitoring m depends on worker education, minimum wage or autonomous work effort. It is interesting to see the effect of monitoring as the exogenous variable. Therefore, we put our job market equilibrium model in a single-equation framework such that

$$D(w, e, r_o, s_o, m_o) = S(w, s_o) \qquad D_w < 0 \quad D_e > 0 \quad D_{s_o} > 0 \quad D_{r_o} < 0$$
$$D_{m_o} < 0 \quad S_w > 0 \quad S_{s_o} < 0$$

where both demand and supply depend on equilibrium wage w and education s_o. However, demand is also positively related to work effort e and negatively to monitoring cost m_o. Thus a higher effort on the part of workers stimulates firms to demand more labor while heavy monitoring increases its cost causing them to reduce their demand for labor. We also introduce a recession parameter r_o, which accounts for the stage in the business cycle that the labor market is in. Thus, in a recessionary period the demand for labor would decrease based on the negative expectations that economic agents have about the future. Rearranging in the form of an implicit function and applying the implicit-function rule, we obtain a number of comparative-static derivatives:

$$D(w,e,r_o,s_o,m_o)-S(w,s_o)=0$$

By implicit differentiation,

$$\frac{\partial w}{\partial s_o}=-\frac{D_{s_o}-S_{s_o}}{D_w-S_w}>0 \qquad \frac{\partial e}{\partial s_o}=-\frac{D_{s_o}-S_{s_o}}{D_e}<0$$

we obtain that the educational level has a positive effect on equilibrium wage and negative on the amount of work effort which is consistent with our previous results. A higher educational level allows the worker to exert less effort, that is, skillful and educated workers need not work as hard as uneducated workers. This effect of the educational level on work effort and wage can further be used to illustrate graphically the relationship between work effort and wage that shows the optimal amount of work effort the worker would exert at a given pay level (See Figure 1). Similar to efficiency wage theory, this analysis presents the optimal work effort – wage combination, with a lower effort-wage ratio for the more skillful, trained and professional workers and a much higher one for the uneducated, poorly trained ones. Thus educated workers receive a high wage for little effort while uneducated ones commit much effort only to receive a lower wage at the end.

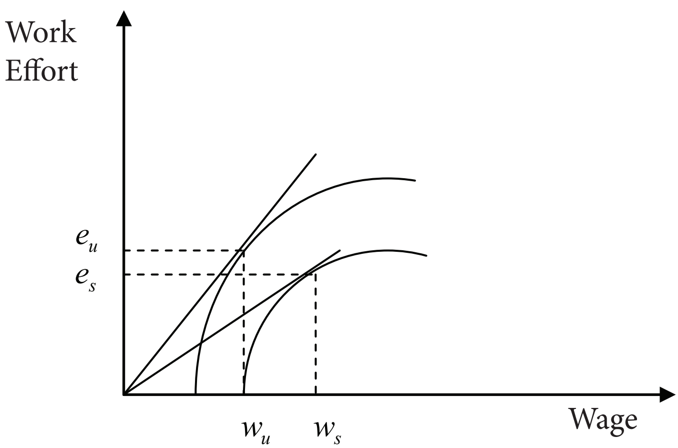

Figure 1: Optimal wage for skilled and unskilled labor

With respect to monitoring and the costs of control on the part of the employer, the effect of monitoring cost is negative on equilibrium wage. This could be explained with the fact that by spending more in control against shirking firms actual pay less in the form of salaries. As an additional cost to the firm monitoring cost reduces its profit which further requires the firm owner to reduce the pay to the worker. Logically, work effort increases with the amount of monitoring. Workers who are monitored more stringently try harder on the job.

$$\frac{\partial w}{\partial m_o} = -\frac{D_{m_o}}{D_w - S_w} < 0 \qquad \frac{\partial e}{\partial m_o} = -\frac{D_{m_o}}{D_e} > 0$$

Finally, with respect to the recession parameter r_o,

$$\frac{\partial w}{\partial r_o} = -\frac{D_{r_o}}{D_w - S_w} < 0 \qquad \frac{\partial e}{\partial r_o} = -\frac{D_{r_o}}{D_e} > 0$$

we see that the effect of the recession is negative on equilibrium wage. As can be expected, due to decreased demand for labor in recessionary times, wage decreases. At the same time, a downturn in the economy increases the work effort for all workers. Since education has an adverse effect on optimal work effort, more skillful and educated workers do not experience the hardships of the recessions so dramatically and their workload does not increase substantively, as shown in Figure 2. However, less educated individuals are adversely affected by the recession in two ways. They now have to invest a lot more effort and receive much less pay than in an economic boom, their effort-wage curve shifting up and increasing their optimal effort-wage ratio. This shift of the curve for less educated, low-skilled workers results merely from the reduction in total market demand for their labor in an economic crisis making uneducated workers more vulnerable in bad economic times.

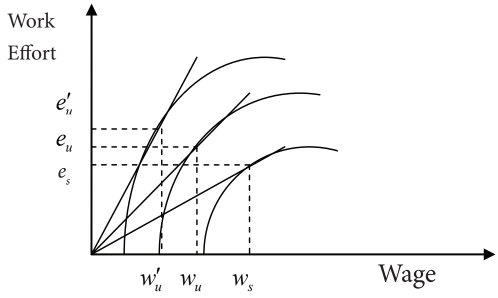

Figure 2: Change of optimal wage in a recession

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Monitoring and profit maximization

If profit is considered a function solely of work effort e, it could be expressed as

$$\pi(e) = pq(e) - C\left[q(e), e\right] - M(e)$$

where price p the firm charges is assumed to be constant[1]. The total production costs C are a function of output and work effort e but ultimately of work effort e since output itself is a function of it. The production function is said to be increasing decreasingly with work effort, that is, $q'(e) = \frac{dq}{de} > 0$ and $q''(e) < 0$. In other words, the firm experiences diminishing returns to work effort. Total production costs are increasing with output, i.e., with the level of work effort where the marginal cost is positive, that is, $C' > 0$. Under the assumption of constant returns to scale marginal cost is constant and equal to average cost. We account for one more effect, the direct effect of work effort on total production costs. It would be negative since greater effort and experience on the part of workers would reduce the firm's costs such that $C_e < 0$. This is the effect of learning by doing, that is, reducing the total production cost due to learning from experience and specialization. Since typical learning curves are convex and negatively sloped, production costs are decreasing slowly with effort, hence, $C_{ee} > 0$. Finally, the total cost of monitoring and control M are directly related to work effort such that the greater the work effort, the less the need for monitoring. Therefore, $M' < 0$. By first-order condition of profit maximization,

$$\pi'(e^*) = pq'(e^*) - C'q'(e^*) - C_e - M'(e^*) = 0$$
$$pq'(e^*) = C'q'(e^*) + C_e + M'(e^*)$$

or at the optimum the employer would produce where his marginal revenue equals the sum of his marginal production and monitoring cost. The second-order condition

$$\pi''(e^*) = pq''(e^*) - C'q''(e^*) - C_{ee} - M''(e^*) < 0$$

1 It could be proven that the results are identical with a nonconstant price $p(q)$. The assumption of constant price, i.e., perfect competition, is made purely for convenience.

proves maximum profit where we know that $C_{ee} > 0$ and $M'' > 0$ that is, monitoring cost decreases slowly with work effort. To see the exact effect monitoring has on work effort we define a specific function of monitoring cost such as $M(e) = \frac{m}{e}$ where m is a positive parameter showing the degree of monitoring by the employer. Thus, $M'(e) = -\frac{m}{e^2} < 0$, and $M''(e) = \frac{2m}{e^3} > 0$.

This function satisfies the conditions for monitoring cost $M(e)$ defined previously, as it decreases asymptotically. Thus, when work effort tends to infinity, that is, $e \to \infty$, there will be no need for monitoring. The profit function of the employer now takes the form

$$\pi(e) = pq(e) - C\left[q(e), e\right] - \frac{m}{e}$$, where

$$\pi'(e^*) = pq'(e^*) - C'q'(e^*) - C_e + \frac{m}{(e^*)^2} = 0$$

gives the optimal work effort to the firm. To prove profit maximization we resort to the second-order condition:

$$\pi''(e^*) = pq''(e^*) - C'q''(e^*) - C_{ee} - \frac{2m}{(e^*)^3} < 0$$

We use implicit differentiation to find the effect of the monitoring parameter m on optimal work effort:

$$\frac{de^*}{dm} = -\frac{1}{(e^*)^2\left[pq''(e^*) - C'q''(e^*) - C_{ee} - \frac{2m}{(e^*)^3}\right]} = -\frac{1}{(p - C')q''(e^*)(e^*)^2 - C_{ee}(e^*)^2 - \frac{2m}{e^*}} > 0$$

The effect is clearly positive since the expression in the denominator is negative for any value of optimal effort.

Monitoring the work effort of two individuals

Let us assume that the firm employs two individuals, one exerting a high work effort e_1 and the other exerting a lower work effort e_2 . Different work efforts do not imply different education and professional skills although high-skilled, highly efficient labor might be associated with more intensive work efforts, good work habits and good discipline. By work effort we assume just the amount of work done at the work place. Let the monitoring cost per unit of effort be $M_i(e_i) = m_{io} - \mu_i e_i$ for each of the two individuals, that is, work effort and monitoring cost are negatively related and the greater the work effort, the lower the monitoring cost. The parameter μ_i is a reward parameter for work discipline and good effort. It shows the degree to which unit monitoring cost is influenced by work effort. There is an autonomous monitoring expenditure on the part of the employer not related to work effort. The parameter m_{io} is exogenous autonomous monitoring by the employer that the worker would experience and be subject to, irrespective of his work effort. Since most workers operating at the same spot would be monitored simultaneously, this parameter is likely to be equal for all. The parameter could also be interpreted as the initial monitoring effort associated with the credentials and achievements of the worker when starting on the job. Therefore, it could be seen as an initial or start-up monitoring. Since some workers are hired as better than others, this parameter is smaller for the former, thus, reducing the overall monitoring cost for those. The per-unit-of-effort monitoring cost of each individual is

$M_1(e_1) = m_{1o} - \mu_1 e_1$ monitoring cost of the first, high-effort worker,

$M_2(e_2) = m_{2o} - \mu_2 e_2$ monitoring cost of the second, low-effort worker.

We expect $m_{1o} < m_{2o}$ or less initial monitoring of the more conscientious, hard working individual. Expressing total profit to the employer,

$$\pi(e_1,e_2) = pq(e_1,e_2) - C\left[q(e_1,e_2),e_1,e_2\right] - M_1(e_1)e_1 - M_2(e_2)e_2 =$$
$$= pq(e_1,e_2) - C\left[q(e_1,e_2),e_1,e_2\right] - (m_{1o} - \mu_1 e_1)e_1 - (m_{2o} - \mu_2 e_2)e_2 =$$
$$= pq(e_1,e_2) - C\left[q(e_1,e_2),e_1,e_2\right] - m_{1o}e_1 + \mu_1 e_1^2 - m_{2o}e_2 + \mu_2 e_2^2$$

By first-order condition,

$$\pi_1 = \frac{\partial \pi}{\partial e_1} = pq_1 - C'q_1 - C_1 - m_{1o} + 2\mu_1 e_1^* = 0$$

$$\pi_2 = \frac{\partial \pi}{\partial e_2} = pq_2 - C'q_2 - C_2 - m_{2o} + 2\mu_2 e_2^* = 0$$

where the marginal cost of production $C' = \frac{\partial C}{\partial q}$ is constant under constant returns to scale and same for both workers. Furthermore, the marginal product of the first worker q_1 is presumably greater than that of the second, less productive worker, q_2 , that is, $q_1 > q_2$. The learning-by-doing effect is measured by the partial derivatives $C_1 = \frac{\partial C}{\partial e_1}$ and $C_2 = \frac{\partial C}{\partial e_2}$, demonstrating how work effort economizes on production costs. Therefore, both these derivatives are negative with the absolute value of the first presumably greater than that of the second. At the same time, the second derivatives C_{11} and C_{22} are positive showing that learning curves are convex and decreasing slowly[2]. The first-order condition gives the optimal work effort for both workers,

$$e_1^* = \frac{m_{1o} - (p - C')q_1 + C_1}{2\mu_1} \qquad e_2^* = \frac{m_{2o} - (p - C')q_2 + C_2}{2\mu_2}$$

where we need $m_{1o} > (p - C')q_1 - C_1$ and $m_{2o} > (p - C')q_2 - C_2$ for positive effort. The optimal work effort depends on the amount of initial monitoring cost, the marginal profit generated by the worker and the effect of learning. Thus, a higher initial monitoring and autonomous control unrelated to work effort forces the worker to work more, while a higher mark-up paired with a higher marginal productivity of labor reduces the optimal work effort. Since the mark-up would be the same for the two types of workers, the marginal product would directly affect the amount of work effort exerted with the more productive worker having a substantially higher marginal product and, therefore, needing to exert much less effort in production. Learning-by-doing has a negative effect on optimal work effort where the first worker is likely to have a higher learning curve. Thus, work experience reduces the optimal amount of work effort still further. Finally, the reward for work discipline reduces work effort. The reward parameter which aims to reward work effort actually reduces it. Thus, the more the employer trusts and rewards the worker, the more likely the worker is to shirk. Finding optimal profit,

2 If there is no cross-learning effect, i.e., workers do not interact and learn from each other, then $C_{12} = C_{21} = 0$

$$\pi_{\max} = \pi(e_1^*, e_2^*) = pq(e_1^*, e_2^*) - C\left[q(e_1^*, e_2^*), e_1^*, e_2^*\right] - M_1 e_1^* - M_2 e_2^* = pq - C - (m_{1o} - \mu_1 e_1^*)e_1^* - (m_{2o} - \mu_2 e_2^*)e_2^* =$$

$$= pq - C - \left\{ m_{1o} - \frac{\mu_1\left[m_{1o} - (p - C')q_1 + C_1\right]}{2\mu_1} \right\} \frac{\left[m_{1o} - (p - C')q_1 + C_1\right]}{2\mu_1} -$$

$$- \left\{ m_{2o} - \frac{\mu_2\left[m_{2o} - (p - C')q_2 + C_2\right]}{2\mu_2} \right\} \frac{\left[m_{2o} - (p - C')q_2 + C_2\right]}{2\mu_2} =$$

$$= pq - C - \frac{\left[m_{1o} + (p - C')q_1 - C_1\right]}{2} \frac{\left[m_{1o} - (p - C')q_1 + C_1\right]}{2\mu_1} -$$

$$- \frac{\left[m_{2o} + (p - C')q_2 - C_2\right]}{2} \frac{\left[m_{2o} - (p - C')q_2 + C_2\right]}{2\mu_2} =$$

$$= pq - C - \frac{m_{1o}^2 - \left[(p - C')q_1 - C_1\right]^2}{4\mu_1} - \frac{m_{2o}^2 - \left[(p - C')q_2 - C_2\right]^2}{4\mu_2} =$$

$$= pq - C - \frac{m_{1o}^2}{4\mu_1} + \frac{\left[(p - C')q_1 - C_1\right]^2}{4\mu_1} - \frac{m_{2o}^2}{4\mu_2} + \frac{\left[(p - C')q_2 - C_2\right]^2}{4\mu_2}$$

Analyzing optimal profit, we see that the more productive worker gives more profit to the employer while costing him less. If $m_{1o} < m_{2o}$, the employer saves money on monitoring the first worker which reduces his overall costs and increases his profit. Since the productivity of the first worker is higher, that is, $q_1 > q_2$ and his learning curve is likely to be steeper, for the same reward parameter the first worker brings more profit to the employer. Representing the result in simple terms with autonomous monitoring and reward assumed to be uniform and no learning effect present, we obtain optimal profit as

$$\pi_{\max} = pq - C - \frac{m_o^2}{2\mu} + \frac{(p - C')^2 q_1^2}{4\mu} + \frac{(p - C')^2 q_2^2}{4\mu}$$

where again the first worker clearly contributes more to profit. Finding optimal monitoring cost for both workers,

$$M_1(e_1^*) = m_{1o} - \frac{\mu_1\left[m_{1o} - (p - C')q_1 + C_1\right]}{2\mu_1} = \frac{m_{1o} + (p - C')q_1 - C_1}{2}$$

$$M_2(e_2^*) = m_{2o} - \frac{\mu_2\left[m_{2o} - (p - C')q_2 + C_2\right]}{2\mu_2} = \frac{m_{2o} + (p - C')q_2 - C_2}{2}$$

Surprisingly monitoring cost is not related to the reward parameter . Therefore, this reward parameter does not play a role in the process of monitoring.

Yet, the reward parameter is essential in increasing total profit since with a positive effort the profit is positively related to μ_i. Thus, the higher the reward parameter, the higher the profit of the firm is. Since the first worker starts at a lower level of autonomous monitoring, he may be subject to lower overall monitoring cost. However, optimal monitoring cost is positively related to the marginal profit generated by each worker. Contrary to the common belief that good workers require less monitoring, the optimal cost of monitoring the first worker, the more productive one, turn out to be higher, not lower. If the initial, autonomous monitoring is the same, that is, $m_{1o} = m_{2o} = m_o$, since both workers are observed simultaneously, then the differential in the monitoring cost of each worker to the employer stems solely from the difference in their marginal products. Thus, interestingly, the more productive worker, not the less productive one, is subject to greater control. This may be due to the fact that the employer is losing essential profit by not observing the effort of the highly productive worker. Poor workers, as well as slow learners, tend to be monitored less while efficient and intelligent workers would be observed strictly. The intuitive explanation is that the employer already knows that he cannot extract much more surplus from the less productive worker. He will be better off monitoring the more productive, skillful one, contributing to his profit more. This explains why in various organizations good workers are forced to work harder than poor workers, although they already provide the highest marginal product of their labor. At the same time, we can conclude that poor workers are like sunk costs to the employer – once hired, they are hard to be dismissed. This is especially true under rigid labor codes or in times of a boom when more output is to be produced and production targets must be met. Since their work potential is low, the employer has just to put up with those and rely on more productive workers. Note that in a recession, when under the threat of losing his job, the less productive worker would increase his marginal product substantially. This would affect the cost and the amount of monitoring him positively.

Both marginal products would also depend on the amount of capital and the efficiency of management. Since the marginal products of both workers would be higher with better and more machinery used, this would increase the optimal monitoring cost of their managers. The more technologically sophisticated the equipment used is, the more monitoring will take place. The more efficient the management and the coordination of productive activities within the firm, the higher the cost of monitoring each individual worker would be.

To equalize the two monitoring cost functions the employer may decide to set $m_{1o} < m_{2o}$, which will compensate for the differential between the productivity of the workers. This reduces the work effort of the first worker still further since his optimal work effort goes down with autonomous monitoring cost. Thus, being more productive, the first worker reduces his effort in three ways: 1) his higher marginal productivity, 2) his lower autonomous monitoring cost, 3) his higher learning curve or stronger learning effect. However with respect to learning it may turn out that the second, less productive worker is learning much faster. While the first might be more productive, the second may actually be the more perceptive one experiencing a higher learning curve. In other words, experienced, qualified workers who are highly productive may not be as fast learning as inexperienced, younger workers. Younger workers who know little but may turn out be fast learners with steep learning curves would meticulously be monitored by the employer as well. Therefore, of the less productive, less skilled workers managers would monitor most strictly the most promising, perceptive and fastest learning. As a second-order condition of profit maximization we form a Hessian with the following second derivatives:

$$\pi_{11} = pq_{11} - C'q_{11} - C_{11} + 2\mu_1 \qquad \pi_{12} = pq_{12} - C'q_{12} - C_{12} = \pi_{21}$$
$$\pi_{22} = pq_{22} - C'q_{22} - C_{22} + 2\mu_2$$

If there is no reward for work effort and workers do not learn from each other, i.e., with reward parameters and cross-learning effects ignored, these become

$$\pi_{11} = pq_{11} - C'q_{11} - C_{11} < 0 \qquad \pi_{12} = pq_{12} - C'q_{12} = \pi_{21}$$
$$\pi_{22} = pq_{22} - C'q_{22} - C_{22} < 0$$

giving rise to the following Hessian

$$|H| = \begin{vmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{vmatrix} = \begin{vmatrix} (p - C')q_{11} - C_{11} & (p - C')q_{12} \\ (p - C')q_{12} & (p - C')q_{22} - C_{22} \end{vmatrix}$$, where

$|H_1| = (p - C')q_{11} - C_{11} < 0$ and

$$|H_2| = |H| = \left[(p - C')q_{11} - C_{11}\right]\left[(p - C')q_{22} - C_{22}\right] - (p - C')^2 q_{12}^2 > 0$$
$$\left[(p - C')q_{11} - C_{11}\right]\left[(p - C')q_{22} - C_{22}\right] > (p - C')^2 q_{12}^2$$

Conclusions

Using a simple job market equilibrium model we find that gross wage is related positively to the educational level of the worker, his initial salary and his autonomous work effort unrelated to any reward or monitoring. Overall work effort is affected positively by autonomous work effort but negatively by education and the initial wage. Monitoring cost depends adversely on minimum or start-up wage, education of the worker and his autonomous effort unrelated to monitoring. Assuming monitoring cost to be exogenous, we find that it reduces gross wage while increasing work effort. In an economic downturn all workers would exert more effort while salaries are likely to decrease. Since poor, less professional, less skilled workers are more vulnerable in a recession, they are likely to experience these negative effects more strongly. Since demand for unskilled labor falls dramatically in an economic crisis such poor workers end up exerting much more effort for a reduced pay. Finally, we find that opposite to common logic, managers monitor strictly more productive workers, fast learners and those starting at a higher autonomous level of monitoring, as those contribute more substantially to the firm's profit.

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